Reidemeister-Franz Torsion as a Homomorphism

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Abstract

Let us denote the Diff-isomorphism classes of n-dimensional non-empty, closed, connected, oriented differentiable manifolds by M_n^{Diff}. An n-manifold M^n is called highly connected if $\pi_1(M^n)=0$ for i=0,…, ⌊n/2⌋-1. So the Diff-isomorphism classes of n-dimensional highly connected differentiable manifolds is given as $M_n^{Diff,hc} = \{M^n \epsilon\ M^{Diff} \mid M^n$ is highly con-nected$\}$. Hence by [1], M_n^{Diff} and $M_n^{Diff,hc}$ are abelian monoids under the connected sum. By [1]-[3], the monoid $M_{2n}^{Diff,hc}$ is a unique factorisation monoid provided that n ≡ 3,5,7 mod 8 except for n=15 or n=31. Suppose that $W^{2n} \epsilon\ M_{2n}^{Diff,hc}$ .Then W^{2n} admits a unique connected sum decomposition into 2n-manifolds that can not be decomposed any further, $W^{2n}=M_1\#M_2\#\cdots\#M_j$. By using such a connected sum decomposition, we prove that Rei- demeister-Franz torsion can be seen as a monoid homomorphism

$$|T_{RF} - |: M_{2n}^{Diff,hc} \rightarrow R^+$$

given by

$$|T_{RF}(W^{2n})| = |T_{RF}(M_1)| \times |T_{RF}(M_2)| \times \cdots \times |T_{RF}(M_j)|.$$

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