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DERIVATION OF THE LORENTZ TRANSFORMATION FROM THE MAXWELL EQUATIONS

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Abstract—The Special Theory of Relativity had been established nearly one century ago to conciliate some seemingly contradictory concepts and experimental results such as the Ether, universal time, contraction of dimensions of moving bodies, absolute motion of the Earth, speed of the light, etc. Hence the fundamental revolutionary formulas of the Theory, i.e., the Lorentz Formulas, had been derived first by Einstein by dwelling on a postulate which stipulated the constancy of the speed of the light. To this end he had first postulated that every reference system has a time proper to itself and then redefined the notions of simultaneity, synchronous clocks, time interval, the length of a rod in a system at rest, the length in a moving system, etc. A second postulate of Einstein, which stated that every physical theory is invariant under the Lorentz transformation, enabled him to claim that the Theory of Electromagnetism is correct but the Newtonian Mechanics has to be re-established. Since then the Theory was *almost* always presented in this way by both Einstein and others except only *a few*. The aim of this paper is to show that the Lorentz formulas can be derived from the Maxwell equations if one pstulates that the total electric charge of an isolated body does not change if it is in motion. To this end one dwells only on the *permanence principle of functional equations*, which is not a physical but purely mathematical concept. Thus, from one side the Special Relativity becomes a *natural* issue (or a part) of the Maxwell Theory and, from the other side, the derivation of the transformation rules pertinent to the electromagnetic field becomes straightforward and easy.

1. INTRODUCTION

As is well known, the Special Theory of Relativity which had been discovered at the beginning of the last century seems to be born as a result of the long discussions on the problem of whether the *absolute motion* of the earth can be detected through observations made on the earth itself. In the earlier papers by Lorentz [1], Poincaré [2] and Einstein [3], published between 1904–1906, as well as in *almost* all papers and books published thereafter the subject was presented in this way and based on the assumption that postulates the constancy of the velocity of the light regardless the situation of its source. To this end Einstein first postulated that every reference system has a time proper to itself and then redefined the notions of simultaneity, synchronous clocks, time interval, the length of a rod in a system at rest, the length in a moving system, etc and then stated two additional postulates which became thereafter the basis of the Theory, namely:

i.) The speed of light in empty space is the same in all inertial frames,

ii.) The laws of physics are the same in all inertial reference frames.

Even today the presentation of the subject is made, *in general*, in this way by dwelling on these postulates (see, for example [4, Ch.40], [5, Ch.3], [6, Ch.1]. See also [3, §2]. It is important to observe that the so-called Lorentz transformation, which constitutes the main basis of the theory, was derived by considering *only* the first of these postulates. As to the second postulate, it was first used by Einstein, together with the Lorentz transformation resulted from the first one, to check if the already established Newtonian Mechanics and Maxwell's Electromagnetism are correct. Thus, from one side it permitted him to *re-establish* the Mechanics, and, from the other side to reveal the rule which interrelates the expressions of the electromagnetic field in different inertial systems. Today it is extensively used to obtain the explicit expressions of solutions to complicated electromagnetic problems, connected with moving bodies, starting from their corresponding expressions supposed to be known in appropriate (rest) systems (see for example [7–18]). So, it is unavoidable in contemporary electrical engineering curriculum (see for example [19]).

Although Einstein and his followers never gave up the light postulate in establishing the Theory of Special Relativity, its role had been subject to some objections since even earlier days of the Theory. For example in 1910 Ignatovski [20] had showed that the light postulate may be replaced by some kinematical assumptions to obtain a one-parameter family of space-time transformation groups under which space-time is invariant. The parameter is of the dimension of velocity and for *any* finite value of the parameter the group is isomorphic to the Lorentz group. Furthermore, the inertial-frame-dependent formalism of Einstein was also objected by some scientists. For example, in 1914 Robb [21] tried to establish the Theory in inertial-frame-independent form. With certain suitable postulates he obtained the geometry of Minkowski space in a purely geometrical manner. More recently Jefimenko [22] had tried to obtain the Lorentz formulas from the retarded potentials connected with the electromagnetic field dwelling on the so-called *effective volume* concept. Since this conecpt, which had been introduced nearly one century ago by Liénard [23], is based on the contraction of length, the starting assumptions of [22] are in fact include the Lorentz formulas themselves.

The aim of the present paper is to show that the Lorentz formulas are *already inherent* in the Maxwell equations and, hence, can be derived from the Maxwell equations if one postulates that the total electric charge of an isolated body does not change when it is in motion. In other words, the special theory of relativity can be thought and taught as a chapter of the Maxwell Theory. This approach makes the Special Theory of Relativity rather natural and easily comprehensible for anybody who is familiar with the Electromagnetism. To this end we solve the *simplest* problem connected with the Maxwell equations in three different coordinate-systems and compare the solutions on the basis of the *permanence principle of functional equations* [24]. The problem consists in finding the electromagnetic field created by a point charge while the coordinate systems in question are as follows:

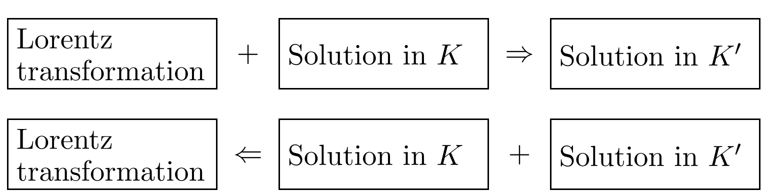
a) A system in which the Maxwell equations are supposed to be valid (an inertial system) while the charged point is in motion with constant (vector) velocity,
b) A system attached to the charged particle,
c) A system which is not inertial.

The discussions to be made in what follows will also enable us to answer clearly the following questions:

a) Are the afore-mentioned postulates (i) and (ii) independent from each other? Is it possible to omit the first one to establish the Special Theory of Relativity?
b) Can anybody *reject* the Lorentz formulas *without rejecting* the Maxwell equations?
c) Is it possible to replace the Lorentz formula by more generalized ones without changing the Maxwell equations?
d) Why the Maxwell equations are correct *only* in inertial systems?

Furthermore, derivation of the transformation rules pertinent to the space coordinates, time as well as the electromagnetic field components become also rather easy by straightforward computations without needing recourse to redefine the aforementioned notions (simultaneity, synchronization of clocks, length of rods, etc), they become immediate results of the Lorentz transformation.

It is also worthwhile to notice that this work was motivated by the fact that the Maxwell equations are invariant under the Lorentz transformation, which claims that by applying this transformation on any solution valid in a system K one obtains the solution valid in another system K' (*a direct problem*). This naturally arises the *inverse problem* as shown schematically in the following figure: Is it possible to obtain the Lorentz transformation by comparing the expressions of a field known in two inertial systems, say K and K'?



The aim of the present work is to show that the answer to the inverse problem is positive if one postulates that the total electric charge of an isolated body does not change if it is in motion. In what follows we will present the solutions pertinent to the aforementioned problems very briefly in order.

2. EXPLICIT EXPRESSIONS OF THE FIELD CREATED BY A POINT CHARGE

2.1. Expressions Valid in a System in Which the Maxwell Equations Are Supposed to Be Valid While the Charged Point Is in Motion with Constant Velocity

Consider an inertial system of reference, say a cartesian coordinate system $Oxyz$, and a point charge of amount Q which makes a uniform motion on a straight-line, say Ox-axis. We postulate that the charge Q of the point remains constant during the motion. If one assumes that at the time $t = 0$ the charge is at the point O and its velocity is equal to v, then the corresponding charge and current densities become

$$\rho = Q\delta(x - vt)\delta(y)\delta(z) \tag{1}$$

and

$$\mathbf{J} = Qv\delta(x - vt)\delta(y)\delta(z)\mathbf{e}_x, \tag{2}$$

respectively. Here $\delta(.)$ denotes the usual Dirac delta distribution. Then from the Maxwell equations written for the vacuum, namely:

$$curl\mathbf{E} + \mu_0\frac{\partial}{dt}\mathbf{H} = 0,\ curl\mathbf{H} - \varepsilon_0\frac{\partial}{dt}\mathbf{E} = \mathbf{J},\ div\mathbf{E} = \rho/\varepsilon_0,\ div\mathbf{H} = 0,$$

where $\mathbf{E}(x, y, z, t)$ and $\mathbf{H}(x, y, z, t)$ are the electric and magnetic fields while the constants ε_0 and μ_0 stand for the permittivity and magnetic permeability of the vacuum, respectively, one writes

$$\mathbf{E} = -gradV - \frac{\partial}{\partial t}\mathbf{A}, \quad \mathbf{H} = \frac{1}{\mu_0}curl\mathbf{A} \tag{3}$$

with

$$V(x, y, z, t) = \frac{Q}{4\pi\varepsilon_0}\iiint \delta\left(\xi - v\left[t - \frac{R}{c}\right]\right)\delta(\eta)\delta(\zeta)\frac{d\xi d\eta d\varsigma}{R} \tag{4}$$

and

$$A_1(x, y, z, t) = \frac{v}{c^2}V(x, y, z, t), \quad A_2 = 0, \quad A_3 = 0. \tag{5}$$

Here c stands for the velocity of the electromagnetic wave in the vacuum ($c = 1/\sqrt{\varepsilon_0\mu_0}$) while $V(x,y,z,t)$ and $\mathbf{A}(x,y,z,t)$ are the usual retarded scalar and vector potentials. As to R, it denotes the distance between the observation point (x,y,z) and the volume element at (ξ,η,ζ), namely:

$$R = \{(x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2\}^{\frac{1}{2}}. \tag{6}$$

In (5) the quantities with sub-indices 1, 2 and 3 signify the cartesian components of a vector, namely: $\mathbf{A} = (A_1, A_2, A_3)$. Notice that the couple of potentials $(\mathbf{A}, V)$ appearing in (3) is *not unique* but to find the field components correctly we can confine ourselves with those given by (4) and (5).

Although an explicit expression of the potential function $V(x,y,z,t)$ defined by (4) is given in many books (see for ex. [25]), for the sake of self-sufficiency we shortly explain how one can compute the triple-integral in (4). By definition of the Dirac distribution, the integrations with respect to η and ζ are immediate and yield

$$V(x,y,z,t) = \frac{Q}{4\pi\varepsilon_0}\int \frac{\delta\left\{\xi - vt + \dfrac{v}{c}\sqrt{(x-\xi)^2+y^2+z^2}\right\}}{\sqrt{(x-\xi)^2+y^2+z^2}}d\xi.$$

To compute this integral we make the substitution

$$\xi - vt + \frac{v}{c}\sqrt{(x-\xi)^2+y^2+z^2} = \lambda \tag{7}$$

which yields

$$d\xi = \frac{d\lambda}{\left[1 - \dfrac{v}{c}\dfrac{x-\xi}{\sqrt{(x-\xi)^2+y^2+z^2}}\right]}$$

and

$$\begin{aligned} V(x,y,z,t) &= \frac{Q}{4\pi\varepsilon_0}\int \frac{\delta(\lambda)}{\sqrt{(x-\xi)^2+y^2+z^2}} \\ &\qquad \frac{d\lambda}{\left[1 - \dfrac{v}{c}\dfrac{x-\xi}{\sqrt{(x-\xi)^2+y^2+z^2}}\right]} \\ &= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{(x-\xi)^2+y^2+z^2} - \dfrac{v}{c}(x-\xi)}\right]_{\lambda=0} \end{aligned} \tag{8}$$

Putting $\lambda = 0$ in (7) one gets, by a straightforward computation, an explicit expression of ξ in terms of x,y,z and t, which reduces (8) into

$$V(x,y,z,t) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{\sqrt{1-v^2/c^2}} \frac{1}{\sqrt{(x-vt)^2/(1-v^2/c^2)+y^2+z^2}}. \tag{9}$$

This expression of the scalar potential, together with (3) and (5), provides us an explicit expression of the electromagnetic field created by the point charge making the uniform motion mentioned above.

2.2. Expressions Valid in a Reference System Attached to the Charged Particle

Consider now a reference system $O'x'y'z'$ whose origin O' moves with the above-mentioned point charge Q such that the axes $O'x'$, $O'y'$ and $O'z'$ are parallel to Ox, Oy and Oz, respectively. It is known that this system is also inertial because, in accordance with the composition rule of velocities to be derived from the Lorentz transformation (*to be obtained later on!*) the acceleration of a point charge is naught in both systems in question if it is zero in one of them. Hence, by our basic assumption the Maxwell equations are also valid in the system $O'x'y'z'$ with the same constants ε_0, μ_0. If we denote all the quantities observed in the system $O'x'y'z'$ with the same letter used above but with an upper sign ($'$), then we write

$$\rho'(x',y',z',t') = Q\delta(x')\delta(y')\delta(z') \tag{10a}$$

and

$$\mathbf{J}'(x',y',z',t') = 0, \tag{10b}$$

which yields

$$V' = \frac{Q}{4\pi\varepsilon_0} \frac{1}{\sqrt{x'^2+y'^2+z'^2}} \tag{11a}$$

and

$$A'_1 = 0, \quad A'_2 = 0, \quad A'_3 = 0. \tag{11b}$$

From these expressions of the potential functions one gets:

$$\mathbf{E}' = -gradV', \quad \mathbf{H}' = 0. \tag{12}$$

3. FOUR-DIMENSIONAL SPACE-TIME OF EINSTEIN

Now assume that the seemingly different expressions (3) and (12) of the same field can be transformed into each other through a *universal* transformation rule. This latter has to transform, from one side (x', y', z', t') to (x, y, z, t) and, from the other side, $(\mathbf{E}', \mathbf{H}')$ to $(\mathbf{E}, \mathbf{H})$. So, if one denotes these transformations by $\mathcal{L}$ and $\mathcal{M}$, respectively, then one writes, for example

$$\mathbf{E} = \mathcal{L}\mathcal{M}_e\{\mathbf{E}'(x', y', z', t'), \mathbf{H}'(x', y', z', t')\}, \quad \forall x', y', z', t' \in (-\infty, \infty) \tag{13}$$

The sub-index (e) appearing in $\mathcal{M}_e$ refers to the electric component to be resulted from $\mathcal{M}(\mathbf{E}', \mathbf{H}')$. Since the motion of the system (x', y', z', t') with respect to (x, y, z, t) consists merely of a translation parallel to the x-axis (or x'-axis), by the definition of the translation (without rotation) one has

$$y' = y, \quad z' = z, \tag{14}$$

which yields also the equality of $r^2 (= z^2 + y^2)$ appearing in (9) with $(r')^2 (= (z')^2 + (y')^2)$ appearing in (11a). That means that both sides of (13) are functions of the quantity $\varsigma = r^2 \equiv (r')^2$. Consider now the analytical continuation of (13), written for $\varsigma > 0$, into the complex ς-plane. In accordance with the *permanence principle of functional equations* [24], the equation (13) is also satisfied in all regions into which the continuations of its left and right hand sides are possible. In other words, (13) is satisfied not only for $\varsigma > 0$ but rather for all complex ς. This requires, first of all, the equivalence of the singularities appearing in both sides. From (9) and (3)–(5) it is obvious that the singularity in the left-hand side consists of the branch singularity at the point $\varsigma = -(x - vt)^2/(1 - v^2/c^2)$. As to the singularity in the right-hand side of (13), before the application of $\mathcal{L}$ it is the branch singularity at $\varsigma = -(x')^2$. Because the transformation $\mathcal{M}$ is supposed to be universal and, hence, independent of the coordinates of the point at which (13) is written. Therefore the operation $\mathcal{M}_e$ taking place in (13) does not change the location of the branch point dictated by (11a)–(12). Hence, the operation $\mathcal{L}$ appearing in (13) has to transform $\varsigma = -(x')^2$ into $\varsigma = -(x - vt)^2/(1 - v^2/c^2)$. Thus one writes

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad y' = y, \quad z' = z, \tag{15}$$

(14) being taken into account. These equations determine the transformation $\mathcal{L}$ completely excepting the expression of t'. This latter will be determined later on.

Now consider (15) in (9) and (11a) to get

$$V = \frac{V'}{\sqrt{1 - v^2/c^2}} \tag{16a}$$

and

$$A_1 = \frac{v}{c^2}\frac{V'}{\sqrt{1-v^2/c^2}}, \quad A_2 = 0, \quad A_3 = 0, \tag{16b}$$

(5) being also taken into account.

Now repeat the analysis made in Section 2 by supposing that the point charge is located at the point O. The results obtained above will be replaced obviously by

$$x = \frac{x' + vt'}{\sqrt{1-v^2/c^2}} \tag{17}$$

and

$$V' = \frac{V}{\sqrt{1-v^2/c^2}}, \tag{18a}$$

$$A'_1 = -\frac{v}{c^2}\frac{V}{\sqrt{1-v^2/c^2}}, \quad A'_2 = 0, \quad A'_3 = 0. \tag{18b}$$

In writing this we *tacitly assume* that if an inertial reference system $O'x'y'z'$ is in motion with velocity $\mathbf{v}$ with respect to the system $Oxyz$, then, inversely, the system $Oxyz$ seems to be in motion with velocity $(-\mathbf{v})$ with respect to $O'x'y'z'$. It is known that this is in accordance with the composition rule of velocities derived from the Lorentz transformation to *be obtained later on*.

From (15) and (17) one gets the complete expression of the transformation $\mathcal{L}$ as follows:

$$\begin{bmatrix} x \\ y \\ z \\ ct \end{bmatrix} = \begin{bmatrix} 1/\sqrt{1-v^2/c^2} & 0 & 0 & (v/c)/\sqrt{1-v^2/c^2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ (v/c)/\sqrt{1-v^2/c^2} & 0 & 0 & 1/\sqrt{1-v^2/c^2} \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ ct' \end{bmatrix}. \tag{19}$$

This is the well-known Lorentz transformation with properties $\mathcal{L}(v)\mathcal{L}(-v) = I$ and $\mathcal{L}^t = \mathcal{L}$, where I stands for the unit (matrix) operator while super-index (t) on $\mathcal{L}^t$ refers to the transpose.

Notice that not quite the same as (19) but rather similar expressions had been obtained by Lorentz before Einstein and Poincaré [1]. But at that time Lorentz had believed that there were some essential differences between (x, y, z, t) and (x', y', z', t') which take place in the transformation formulas, the latter being only *auxiliary mathematical quantities* [26]. Therefore, according to the opinion of Lorentz, t' should not be thought as the correct time in the system $O'x'y'z'$ [26]. But Einstein bravely claimed that (19) is not only a mathematical receipt to transform the expressions of the electromagnetic field known in $O'x'y'z'$ into expressions valid in $Oxyz$, but rather general and *associated with the reference systems* $Oxyz$ (including t) and $O'x'y'z'$ (including t') themselves. In other words, from physical phenomena point of view the space is a four-dimensional variety.

Consider finally the relations (16a, 16b) and (18a, 18b), which yields:

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ V/c \end{bmatrix} = \begin{bmatrix} 1/\sqrt{1-v^2/c^2} & 0 & 0 & (v/c)/\sqrt{1-v^2/c^2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ (v/c)/\sqrt{1-v^2/c^2} & 0 & 0 & 1/\sqrt{1-v^2/c^2} \end{bmatrix} \begin{bmatrix} A'_1 \\ A'_2 \\ A'_3 \\ V'/c \end{bmatrix}. \tag{20}$$

From this one concludes that the quadruple $(A_1, A_2, A_3, V/c)$ is also transformed as the four-vector (x, y, z, ct). Therefore, the *particular* couple of potential functions given by (4) and (5) is a vector-valued function of the four-dimensional space with transform given by (11a)–(11b). It is important to remark here that although the relation (20) was derived for the *particular* field defined in Sec. 2, it is quite *general* (for proof see the *remark* to be made at the end of Sec. 4 below).

4. TRANSFORMATION RULES OF THE FIELD COMPONENTS

The transformation rules (19) and (20), with the relations (3), will permit us to reveal the explicit expression of the transformation $\mathcal{M}$ rather easily. Indeed, from the first equation one writes

$$\begin{aligned} E_1 &= -\frac{\partial V}{\partial x} - \frac{\partial A_1}{\partial t} \\ &= -\frac{1}{\sqrt{1-v^2/c^2}}\left[\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right]\frac{1}{\sqrt{1-v^2/c^2}}[vA'_1 + V'] \\ &\qquad -\frac{1}{\sqrt{1-v^2/c^2}}\left[-v\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}\right]\frac{1}{\sqrt{1-v^2/c^2}}\left[A'_1 + \frac{v}{c^2}V'\right] \\ &= -\frac{\partial V'}{\partial x'} - \frac{\partial A'_1}{\partial t'} \\ &= E'_1, \end{aligned} \tag{21a}$$

$$\begin{aligned} E_2 &= -\frac{\partial V}{\partial y} - \frac{\partial A_2}{\partial t} \\ &= -\frac{\partial}{\partial y'}\frac{1}{\sqrt{1-v^2/c^2}}[vA'_1 + V'] - \frac{1}{\sqrt{1-v^2/c^2}}\left[-v\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}\right]A'_2 \end{aligned}$$

$$= -\frac{1}{\sqrt{1-v^2/c^2}}\left[\frac{\partial V'}{\partial y'} + \frac{\partial A'_2}{\partial t'}\right] + \frac{v}{\sqrt{1-v^2/c^2}}\left[\frac{\partial A'_2}{\partial x'} - \frac{\partial A'_1}{\partial y'}\right]$$

$$= \frac{1}{\sqrt{1-v^2/c^2}}[E'_2 + vB'_3] \tag{21b}$$

and

$$E_3 = \frac{1}{\sqrt{1-v^2/c^2}}[E'_3 - vB'_2]. \tag{21c}$$

Quite similarly, from $\mathbf{B} = \mu_0\mathbf{H} = curl\mathbf{A}$ one gets also

$$B_1 = B'_1, \tag{22a}$$

$$B_2 = \frac{1}{\sqrt{1-v^2/c^2}}\left[B'_2 - \frac{v}{c^2}E'_3\right] \tag{22b}$$

$$B_3 = \frac{1}{\sqrt{1-v^2/c^2}}\left[B'_3 + \frac{v}{c^2}E'_2\right]. \tag{22c}$$

The rules pertinent to $\mathbf{D}$ and $\mathbf{H}$ are obtained directly from (21a)–(22c) by considering their definitions, namely: $\mathbf{D} = \varepsilon_0\mathbf{E}$ and $\mathbf{H} = \mathbf{B}/\mu_0$, which yields

$$D_1 = D'_1, D_2 = \frac{1}{\sqrt{1-v^2/c^2}}[D'_2 + \frac{v}{c^2}H'_3], D_3 = \frac{1}{\sqrt{1-v^2/c^2}}[D'_3 - \frac{v}{c^2}H'_2], \tag{23}$$

and

$$H_1 = H'_1, H_2 = \frac{1}{\sqrt{1-v^2/c^2}}[H'_2 - vD'_3], H_3 = \frac{1}{\sqrt{1-v^2/c^2}}[H'_3 + vD'_2] \tag{24}$$

Finally, as to the rules for $\mathbf{J}$ and ρ, they can be obtained from the equations $\mathbf{J} = curl\mathbf{H} - \partial\mathbf{D}/\partial t$ and $\rho = div\mathbf{D}$ by considering also (23) and (24). Indeed, by repeating the direct computations that were used to obtain (21a, 21b), one gets

$$J_1 = \frac{\partial}{\partial y}H_3 - \frac{\partial}{\partial z}H_2 - \frac{\partial}{\partial t}D_1$$

$$= \frac{1}{\sqrt{1-v^2/c^2}}\left[\frac{\partial}{\partial y'}H'_3 - \frac{\partial}{\partial z'}H'_2 - \frac{\partial}{\partial t'}D'_1\right]$$

$$+ \frac{v}{\sqrt{1-v^2/c^2}}\left[\frac{\partial}{\partial x'}D'_1 + \frac{\partial}{\partial y'}D'_2 + \frac{\partial}{\partial z'}D'_3\right]$$

$$= \frac{1}{\sqrt{1-v^2/c^2}}J'_1 + \frac{v}{\sqrt{1-v^2/c_0^2}}div\mathbf{D}'$$

$$= \frac{1}{\sqrt{1-v^2/c^2}}[J_1' + \rho' v], \tag{25a}$$

$$J_2 = J_2', \tag{25b}$$
$$J_3 = J_3' \tag{25c}$$

and

$$\begin{aligned} \rho &= div\mathbf{D} \\ &= \frac{1}{\sqrt{1-v^2/c^2}}\left[\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right] D_1' \\ &\quad + \frac{\partial}{\partial y'}\frac{1}{\sqrt{1-v^2/c^2}}\left[D_2' + \frac{v}{c^2}H_3'\right] + \frac{\partial}{\partial z'}\frac{1}{\sqrt{1-v^2/c^2}}\left[D_3' - \frac{v}{c^2}H_2'\right] \\ &= \frac{1}{\sqrt{1-v^2/c^2}}\left[\rho' + \frac{v}{c^2}J_1'\right]. \end{aligned} \tag{26}$$

It is obvious from (25a, 25b, 25c) and (26) that the quantity $(J_1, J_2, J_3, c\rho)$ is a four-vector which is transformed according to the rule (20).

A remark. Once the relations (21a)–(24), which transform the expressions of a field valid in K into those valid in K', were established, then by considering (21a) and (22a) one can easily show that the equation (20) is valid for all fields. However it needs some more clarification because the couple of potentials pertinent to a given field is not unique. For example, it is not permitted to replace the couples ($\boldsymbol{A}$, V/c) and ($\boldsymbol{A'}$, V/c), which appear in the left and right-hand sides of (20), by *any* possible couples of potentials. The meaning of (20) is that if *any* couple of potentials ($\boldsymbol{A'}$, V/c), valid in the system K', is inserted in the right-hand side of (20), then the left-hand side becomes one of the possible couples of potentials which are valid in the system K.

5. MAXWELL EQUATIONS AND NON-INERTIAL REFERENCE SYSTEMS

Now we want to show that the Maxwell equations are not valid in non-inertial systems, and, therefore it is not meaningful to try to extend the Special Relativity Theory to non-inertial systems in order to obtain solutions of electromagnetic problems connected with accelerated bodies from the solutions of static problems. To this end let us consider a point charge Q located at the origin of a non-inertial reference system $O''x''y''z''$ which makes a non-uniform motion with respect to the inertial system $Oxyz$. If the velocity of the point charge is denoted by $\mathbf{v}(t)$, then, by solving the Maxwell equations in the system $Oxyz$ one gets (following expressions can be obtained through a straightforward extension of the approach adopted in Section 2 above (Cf [25]):

$$V(x,y,z,t) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\{\Re - \vec{\Re}.\vec{\mathbf{v}}/c\}} \tag{27a}$$

and

$$\mathbf{A}(x,y,z,t) = \frac{\mathbf{v}}{c^2}V(x,y,z,t), \tag{27b}$$

where c stands, as usual, $1/\sqrt{\varepsilon_0\mu_0}$ while $\Re$, $\vec{\Re}$ and $\vec{v}$ signify the following retarded quantities:

$$\begin{aligned}
\vec{\Re} &= \left[x - \alpha\left(t - \frac{\Re}{c}\right)\right]\mathbf{e}_x + \left[y - \beta\left(t - \frac{\Re}{c}\right)\right]\mathbf{e}_y + \left[z - \gamma\left(t - \frac{\Re}{c}\right)\right]\mathbf{e}_z \\
\Re &= \sqrt{\left[x - \alpha\left(t - \frac{\Re}{c}\right)\right]^2 + \left[y - \beta\left(t - \frac{\Re}{c}\right)\right]^2 + \left[z - \gamma\left(t - \frac{\Re}{c}\right)\right]^2} \\
\vec{v} &= \frac{d}{dt}\alpha\left(t - \frac{\Re}{c}\right)\mathbf{e}_x + \frac{d}{dt}\beta\left(t - \frac{\Re}{c}\right)\mathbf{e}_y + \frac{d}{dt}\gamma\left(t - \frac{\Re}{c}\right)\mathbf{e}_z.
\end{aligned}$$

Here the functions $(\alpha(t), \beta(t), \gamma(t))$ describe the motion of the point charge. In other words, the trajectory of the point charge consists of the curve

$$x = \alpha(t), \quad y = \beta(t), \quad z = \gamma(t). \tag{28}$$

The field expressions to be derived from the potentials (27a, 27b) are correct for all continuously differentiable functions $(\alpha(t), \beta(t), \gamma(t))$ because the Maxwell equations are assumed to be valid in the inertial reference system $Oxyz$.

If the Maxwell equations *were also valid* in the non-inertial system $O''x''y''z''$, then, by solving them there we could get again (11a) and (11b) with the only difference that ($'$) is replaced now by ($''$), namely:

$$V'' = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\sqrt{x''^2 + y''^2 + z''^2}}, \quad \mathbf{A}'' = 0. \tag{29}$$

Then, by comparing (27a) with (29) we would obtain an equation of the following form, which replaces (15) in this case:

$$L\left\{\sqrt{\left[x - \alpha\left(t - \frac{\Re}{c}\right)\right]^2 + \left[y - \beta\left(t - \frac{\Re}{c}\right)\right]^2 + \left[z - \gamma\left(t - \frac{\Re}{c}\right)\right]^2}\right\}$$

$$\equiv \sqrt{x''^2 + y''^2 + z''^2}. \quad (30)$$

Here L denotes the transformation which changes (x, y, z, t) into (x'', y'', z'', t''). Since L is supposed to be a law of the nature, it will enforce us to re-establish the *mechanics* just as the Lorentz transformation did one century ago. Obviously this new form of the mechanics, which would have been affected by the particular functions $(\alpha(t), \beta(t), \gamma(t))$, would not be universal as already established relativistic mechanics. From this we conclude that it is not possible to pretend the validity of the Maxwell equations in the non-inertial reference system $O''x''y''z''$.

6. CONCLUSIONS AND CONCLUDING REMARKS

As it was pointed out above, the aim of this paper is to present an alternate approach to arrive at the Lorentz transformation. In addition to a purely mathematical concept (i.e., the permanence principle of functional equations), it dwells on the following physical assumptions:

i) the Maxwell equations describe correctly the electromagnetic phenomenon (in the vacuum) in all inertial reference systems for all kind of source distributions,

ii) the total electric charge of a body does not change if it is in motion,

iii) if an inertial system slides without rotation, with respect to a second inertial system, in the direction of x-axis, then during the motion the y and z coordinates of all points remain constant,

iv) if an inertial reference system moves with a constant velocity $\mathbf{v}$ with respect to a second inertial system, then, conversely, the latter seems to be in motion with velocity $(-\mathbf{v})$ with respect to the first.

The assumption (i) consists of the second postulate of Einstein while (ii)–(iv) are quite reasonable and, also, are in accordance with all the formulas to be derived from the Lorentz transformation.

The analysis made above shows that the essentials of the Lorentz transformation are all (except the invariance of the total charge) implicitly existing in the Maxwell equations put forth in 1873, namely thirty years before the Einstein's famous paper. It is rather curious that during a long period of fervent discussions that preceded the Special Theory of Relativity nobody tried to resort to it to bring an explanation to the contraction of dimensions of a moving body. The concept of four-dimensional space-time and its transformation rules could also be revealed very early without waiting to see the results of the experiments due to Michelson and Morley.

To conclude this section we would like to notice also some historical points.

(i) The so-called Lorentz Transformation was derived (mathematically) not by Lorentz but simultaneously by Poincaré and Einstein in 1905. This fact was confirmed by Lorentz himself in 1921 [26]. The formulas written by Lorentz in 1895 were ad-hoc expressions introduced to explain the experimental results obtained by Michelson and Morley nearly one decade ago (between 1881 and 1887).

(ii) The approach adopted by Poincaré [2] was based on the concept of contraction of dimensions of bodies under motions in the Ether, which was introduced by Lorentz [1] while Einstein [3] had derived the formulas by considering only the travel of the light in opposite directions.

(iii) We owe also the term *relativity postulate* to Poincaré. He introduced this term to indicate the fact that "the impossibility of revelation by experiments the absolute motion of the Earth" is a general law of the Nature [27].

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