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**DESIGN OF REED-SOLOMON AND CONVOLUTIONAL CODED OFDM
SYSTEM AND PERFORMANCE ANALYSIS**

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Ufuk Bal

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Approval of the Institute of Science and Engineering

Prof. Dr. Endal Panayirci

(Title and Name)
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.



Prof. Dr. Endal Panayirci
(Title and Name)
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

(Title and Name)
Co-Supervisor

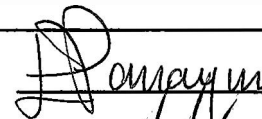
Prof. Dr. Endal Panayirci
(Title and Name)
Supervisor

Examining Committee of Members

Prof. Dr. Endal Panayirci (IŞIK)

Prof. Dr. Ahmet Akşen (IŞIK)

Prof. Dr. Ümit Aygözü (İTÜ)





ABSTRACT

DESIGN OF REED-SOLOMON AND CONVOLUTIONAL CODED OFDM SYSTEM AND PERFORMANCE ANALYSIS

Bal, Ufuk

In this study, OFDM and error-correcting codes, which have an important place in digital communication systems, have been studied. Literature research has been made on OFDM and error-correcting codes like Reed-Solomon and convolutional codes. Reed-Solomon and convolutional coded OFDM system has been designed. Furthermore, system's performance analysis has been made.

OFDM is a special case of multicarrier transmission. The basic principle of OFDM is to split a high-rate datastream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers. One of the main reasons to use OFDM is to increase the robustness against frequency selective fading or narrowband interference. In a single carrier system, a single fade or interferer can cause the entire link to fail, but in a multicarrier system, only a small percentage of the subcarriers will be affected. Error correcting codes can then be used to correct the errors.

In error correcting, combining convolutional and block codes in a concatenated code is particularly powerful technique. In this study Reed-Solomon codes are used as an outer code and convolutional codes are used as an inner code.

ÖZET

REED-SOLOMON VE KONVOLÜSYONEL KODLU DİKGEN FREKANS BÖLMELİ ÇOĞULLAMA(OFDN) SİSTEMİ'NİN TASARIMI VE PERFORMANS ANALİZİ

Bal, Ufuk

Bu çalışmada sayısal haberleşme sistemlerinde önemli yeri olan OFDM ve hata düzelten kodlar incelenmiştir.OFDN ve hata düzelten kodlama çeşitlerinden Reed-Solomon ve Konvolüsyonel kodlara yönelik bir literatür araştırması ile Reed-Solomon ve Konvolüsyonel kodlu OFDM sistem tasarımı yapılmıştır.Ayrıca bu sistemin performans analizi yapılmıştır.

OFDM çok-taşıyıcılı iletimin özel bir durumudur. OFDM'in temel prensibi yüksek orandaki bilgi dizisini alttaşıyıcılar üzerinden aynı anda iletilen daha düşük oranda bilgi dizilerine ayırmaktır. OFDM kullanmanın önemli nedenlerinden biri frekans seçmeli bozulmaya veya darband girişimine karşı dayanıklılığı arttırmasıdır.Tek taşıyıcılı bir sistemde, tek bir bozulma veya girişim bütün hattı etkileyebilir, fakat çok taşıyıcılı bir sistemde, sadece küçük bir oranı etkilenir.Daha sonrada hata düzeltim kodlaması, bu hataları düzeltmek için kullanılabilir.

Hata düzeltmede Konvolüsyonel ve blok kodların birlikte kullanımı çok etkili bir yöntemdir.Bu çalışmada dış kod olarak Reed-solomon ve iç kod olarak Konvolüsyonel kodları kullanılmıştır.

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1 INTRODUCTION

Multi-carrier modulation, in particular Orthogonal Frequency Division Multiplexing (OFDM), has been successfully applied to a wide variety of digital communications applications over the past several years.

In almost all applications of multi-carrier modulation, satisfactory performance cannot be achieved without the addition of some form of coding. In wireless systems subjected to fading, extremely high signal-to-noise ratios are required to achieve reasonable error probability. In addition, interference from other wireless channels is frequently severe. On wireline systems, large constellation sizes are commonly employed to achieve high bit rates. Coding in this case is essential for achieving the highest possible rates in the presence of crosstalk and impulsive and other interference.

So, here first the basics of OFDM are presented. In addition to basics of OFDM, the subsystems of an OFDM implementation are described, such as synchronization and coding. Then error correcting codes, specially Reed-Solomon codes, are described in detail. At last, RS and Convolutional coded OFDM system's simulation results are given. The system's details are given in the 7th chapter.

2 OFDM

2.1 Evolution of OFDM

The use of Frequency Division Multiplexing (FDM) goes back over a century, where more than one low rate signal, such as telegraph, was carried over a relatively wide bandwidth channel using a separate carrier frequency for each signal. To facilitate separation of the signals at the receiver, the carrier frequencies were spaced sufficiently far apart so that the signal spectra did not overlap. Empty spectral regions between the signals assured that they could be separated with readily realizable filters. The resulting spectral efficiency was therefore quite low.

Instead of carrying separate messages, the different frequency carriers can carry different bits of a single higher rate message. The source may be in such a parallel format, or a serial source can be presented to a serial-to-parallel converter whose output is fed to the multiple carriers. Such a parallel transmission scheme can be compared with a single higher rate serial scheme using the same channel. The parallel system, if built straightforwardly as several transmitters and receivers, will certainly be more costly to implement. Each of the parallel sub-channels can carry a low signalling rate, proportional to its bandwidth. The sum of these signalling rates is less than can be carried by a single serial channel of that combined bandwidth because of the unused guard space between the parallel subcarriers. On the other hand, the single channel will be far more susceptible to inter-symbol interference. This is because of the short duration of its signal elements and the higher distortion produced by its wider frequency band, as compared with the long duration signal elements and narrow bandwidth in sub-channels in the parallel system. Before the development of equalization, the parallel technique was the preferred means of achieving high rates over a dispersive channel, in spite of its high cost and relative bandwidth inefficiency. An added benefit of the parallel technique is reduced susceptibility to most forms of impulse noise.

The first solution of the bandwidth efficiency problem of multi-tone transmission (not the complexity problem) was probably the “Kineplex” system. The Kineplex system was developed by Collins Radio Co. For data transmission over an H.F. radio channel subject to severe multi-path fading. In that system, each of 20 tones is modulated by differential 4-PSK without filtering. The spectra therefore of the $\sin(kf)/f$ shape and strongly overlap. However, similar to modern OFDM, the tones are spaced at frequency intervals almost equal to the signalling rate and are capable of separation at the receiver.

The reception technique is shown in Figure 2.1. Each tone is detected by a pair of tuned circuits. Alternate symbols are gated to one of the tuned circuits, whose signal is held for the duration of the next symbol. The signals in the two tuned circuits are then processed to determine their phase difference, and therefore the transmitted information. The older of the two signals is then quenched to allow input of the next symbol. The key to the success of the technique is that the time response of each tuned circuit to all tones. Other than the one to which it is tuned, goes through zero at the end of the gating interval, at which point that interval is equal to the reciprocal of the frequency separation between tones. The gating time is made somewhat shorter than the symbol period to reduce inter-symbol interference, but efficiency of 70% percent of the Nyquist rate is achieved. High performance over actual long H.F. channels was obtained, although at a implementation cost. Although fully transistorized, the system required two large bays of equipment.

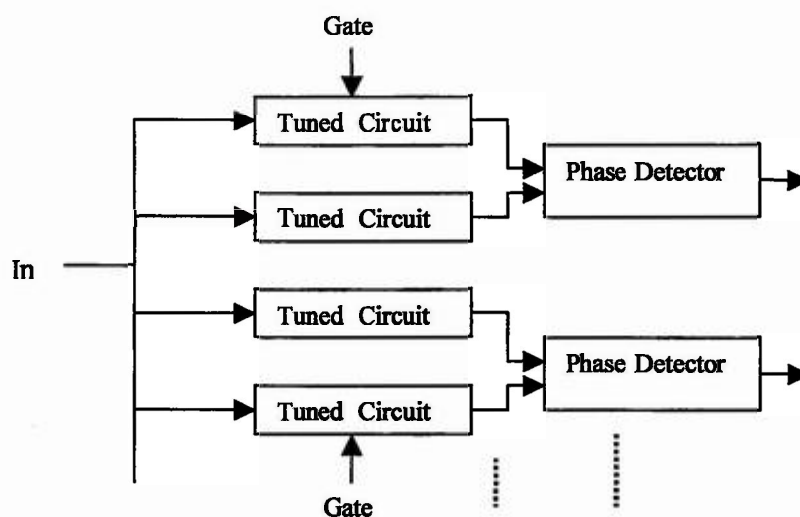


Figure 2.1 The Collins Kineplex receiver

A subsequent multi-tone system was proposed using 9-point QAM constellations on each carrier, with correlation detection employed in the receiver. Carrier spacing equal to the symbol rate provides optimum spectral efficiency. Simple coding in the frequency domain is another feature of this scheme. The above techniques do provide the orthogonality needed to separate multi-tone signals spaced by the symbol rate. However the $\text{sinc}(kf)/f$ spectrum of each component has some undesirable properties. Mutual overlap of a large number of subchannel-spectra is pronounced. Also, spectrum for the entire system must allow space above and below the extreme tone frequencies to accommodate the slow decay of the sub-channel spectra. For these reasons, it is desirable for each of the signal components to be bandlimited so as to overlap only the immediately adjacent sub-carriers, while remaining orthogonal to them.

The major contribution to the OFDM complexity problem was the application of the Fast Fourier Transform (FFT) to the modulation and demodulation processes. Fortunately, this occurred at the same time digital signal processing techniques were being introduced into design of modems. The technique involved assembling the input information into blocks of N complex numbers, one for each sub-channel. An inverse FFT is performed on each block, and the resultant transmitted serially. At the receiver, the information is recovered by performing an FFT on the received block of signal samples. This form of OFDM is often referred to as Discrete Multi-Tone (DMT). The spectrum of the signal on the line is identical to that of N separate QAM signals, at N frequencies separated by the signalling rate. Each such QAM signal carries one of the original input complex numbers. The spectrum of each QAM signal is of the form $\text{sinc}(kf)/f$, with nulls at the center of the other sub-carriers, as in the earlier OFDM systems.

Overlap of consecutive transmitted blocks is a problem, we can solve by using cyclic prefix. Another issue is how to transmit the sequence of complex numbers from the output of the inverse FFT over the channel.

The process is straightforward if the signal is to be further modulated by a modulator with I and Q inputs.

Otherwise, it is necessary to transmit real quantities. This can be accomplished by first appending the complex conjugate to the original input block. A $2N$ -point inverse FFT now yields $2N$ real numbers to be transmitted per block, which is equivalent to N complex numbers.

The most significant advantage of this DMT approach is the efficiency of the FFT algorithm. An N -point FFT requires only on the order of $N \log N$ multiplications, rather than N^2 as in a straightforward computation. The efficiency is particularly good when N is a power of 2, although that is not generally necessary. Because of the use of the FFT, a DMT system typically requires fewer computations per unit time than an equivalent single channel system with equalization. An overall cost comparison between the two systems is not as clear, but the costs should be approximately equal in most cases.

Over the last 20 years or so, OFDM techniques and, in particular, the DMT implementation, has been used in a wide variety of applications. Several OFDM voiceband modems have been adopted as the standard for the Asymmetric Digital Subscriber Line (ADSL), which provides digital communication at several Mb/s from a telephone company central office to a subscriber, and a lower rate in the reverse direction, over a normal twisted pair of wires in the loop plant.

OFDM has been particularly successful in numerous wireless applications, where its superior performance in multi-path environments is desirable. Wireless receivers detect signals distorted by time and frequency selective fading. OFDM in conjunction with proper coding and interleaving is a powerful technique for combating the wireless channel impairments that a typical OFDM system might face.

2.2 OFDM Basics

The basic principle of OFDM is to split a high-rate datastream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers. Because the symbol duration increases for the lower rate parallel subcarriers, the relative amount of dispersion in time caused by multipath delay spread is decreased.

An OFDM signal consists of a sum of subcarriers that are modulated by using phase shift keying (PSK) or quadrature amplitude modulation (QAM). If d_i are the complex QAM symbols N_s is the number of subcarriers, T the symbol duration, and f_c the carrier frequency, then one OFDM symbol starting at $t=t_s$ can be written as

$$s(t) = \text{Re} \left\{ \sum_{i=-N_s/2}^{i=N_s/2-1} d_{i+N_s/2} \exp(j2\pi(f_c - (i+0.5)/T)(t-t_s)) \right\}, t \leq t_s \leq t_s+T$$

$$s(t) = 0, t < t_s \wedge t > t_s+T$$

(2.1)

In the literature, often the complex baseband notation is used. The real and imaginary parts correspond to the in-phase and quadrature parts of the OFDM signal which have to be multiplied by a cosine and sine of the desired carrier frequency to produce the final OFDM signal.

$$s(t) = \sum_{i=-N_s/2}^{i=N_s/2-1} d_{i+N_s/2} \exp(j2\pi(i/T)(t-t_s)), t \leq t_s \leq t_s+T$$

$$s(t) = 0, t < t_s \wedge t > t_s+T$$

(2.2)

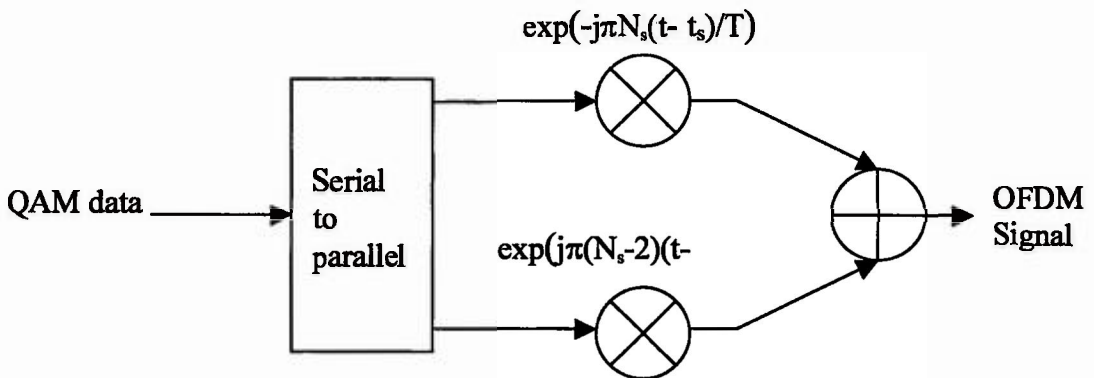


Figure 2.2 OFDM Modulator

